Why the Bradley aberration cannot be used to measure absolute speeds. A comment.

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In a recent article in this journal [1], Sardin proposed to use the Bradley aberration of light for the construction of a speedometer capable of measuring absolute speeds. The purpose of this comment is to show that the device would not work.

Stellar aberration appears to be independent of the velocity of the star observed. This fact, even though explained long ago [2], has remained a source of continuing confusion. After all, it seems difficult to reconcile with the relativity principle. Indeed, if the arguments of [1] could be upheld, they would lead to a rejection of this principle. To clarify the issue a bit, I will discuss different concepts of aberration and their relation to the Bradley aberration.

Roughly speaking, aberration is the difference between the observed and "true" angular positions of a star, caused by the motion of either the observer or the star. A problem with this definition is that there is no such thing as a true position. Positions have been observer dependent since the days of Newton. In order to get a workable definition of *true aberration*, a distinguished observer is needed determining the frame in which to measure the true position of a star. Clearly, the most natural choice would be an observer *at rest* with respect to the star, because she will always see it at the same angle. Her frame of reference is the same as that of the star, so I will consider just the frames of observer and star. It is then easy to calculate the so-defined true aberration for various situations. For simplicity, the calculations will be done with the velocities of the star and the observer(s) parallel to a prescribed line taken as x axis.

First assume the star to move at velocity $-\mathbf{V}$ (in a certain frame Σ') and the observer to be at rest. Let a light ray emitted at an angle θ (see Fig. 1, left) in the star's frame Σ hit the observer's eye. What is the observed angle θ'?

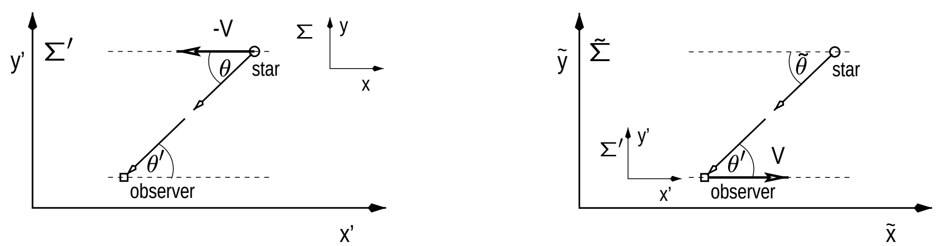

FIG. 1. Aberration for moving source (left) and for moving observer (right).

The answer is provided by the relativistic addition theorem for velocities:

$$\mathbf{u}' = \mathbf{u}'_{\parallel} + \mathbf{u}'_{\perp} = \frac{\mathbf{u}_{\parallel} - \mathbf{V}}{1 - \mathbf{V}\mathbf{u}/c^2} + \frac{\mathbf{u}_{\perp}}{\gamma(1 - \mathbf{V}\mathbf{u}/c^2)}\,, \tag{1}$$

where $\mathbf{u}$ is the velocity in Σ, $\mathbf{u}_{\parallel} \equiv \mathbf{V}(\mathbf{V}\mathbf{u})/V^2$ and $\mathbf{u}_{\perp} \equiv \mathbf{u} - \mathbf{u}_{\parallel}$ are its components parallel and perpendicular to $\mathbf{V}$, respectively, $\mathbf{u}'$, $\mathbf{u}'_{\parallel}$, $\mathbf{u}'_{\perp}$ are the corresponding quantities in Σ' and $\gamma = (1-V^2/c^2)^{-1/2}$. Note that while the *speed* of light is the same in the two frames, its *velocity* (vector) is not. In Σ, we have $\mathbf{u} = -c(\cos\theta\mathbf{e}_x + \sin\theta\mathbf{e}_y)$, so we obtain $\mathbf{u}' = -c(\cos\theta'\mathbf{e}_x +$

$\sin\theta'\mathbf{e}_y)$, with $\cos\theta' = (\cos\theta + V/c)/[1 + (V/c)\cos\theta]$ and $\sin\theta' = \sin\theta/[\gamma(1 + (V/c)\cos\theta)]$, which can be simplified, using $\tan x/2 = \sin x/(1 + \cos x)$, to yield the well-known result

$$\tan\frac{\theta'}{2} = \tan\frac{\theta}{2}\sqrt{\frac{c - V}{c + V}}\,. \tag{2}$$

Consider now the situation (Fig. 1, right) where the observer moves at $\mathbf{V}$ in some system $\tilde{\Sigma}$, while the star is at rest (hence $\tilde{\Sigma} = \Sigma$). If the angle observed in Σ' is θ' again, what will be its value in $\tilde{\Sigma}$? One finds, using the addition theorem with $\mathbf{V}$ instead of $-\mathbf{V}$: $\tan\tilde{\theta}/2 = \tan\theta'/2\,\sqrt{(c+V)/(c-V)}$. Hence $\tilde{\theta} = \theta$, and the *true aberration* is the same for the two situations. It depends on the *relative* velocities of star and observer only.

However, with an *a priori* unknown velocity of the star, an observer at rest with respect to it is not normally handy. Therefore what is *measured* is not the true aberration, in general. Let us then define as *relative aberration* the difference in angles observed by two arbitrary observers moving at different velocities $\mathbf{V_1}$ and $\mathbf{V_2}$ and looking at the same star (the moment they meet). From (2), we have $\tan\theta_1/2 = \tan\theta/2\sqrt{(c - V_1^s)/(c + V_1^s)}$ and $\tan\theta_2/2 = \tan\theta/2\sqrt{(c - V_2^s)/(c + V_2^s)}$, where V_1^s and V_2^s are the velocities of the observers *relative* to the star. θ may be eliminated to obtain $\tan\theta_2/2 = \tan\theta_1/2\sqrt{(c - V_2^s)(c + V_1^s)/(c + V_2^s)(c - V_1^s)}$. Using the velocity addition theorem to express $V_{1/2}^s$ by $V_{1/2}$ and the unknown velocity W of the star, $V_{1/2}^s = (V_{1/2} - W)/(1 - V_{1/2}W/c^2)$, we find $(c + V_{1/2}^s)/(c - V_{1/2}^s) = [(c - W)(c + V_{1/2})]/[(c + W)(c - V_{1/2})]$ and thus

$$\tan\frac{\theta_2}{2} = \tan\frac{\theta_1}{2}\sqrt{\frac{(c-W)(c+V_1)}{(c+W)(c-V_1)}\frac{(c+W)(c-V_2)}{(c-W)(c+V_2)}} = \tan\frac{\theta_1}{2}\sqrt{\frac{c - V_{21}}{c + V_{21}}}\,, \tag{3}$$

where $V_{21} = (V_2 - V_1)/(1 - V_1V_2/c^2)$ is the *relative* velocity between the *two observers.* The speed W of the star cancels out of the formula! Of course, we could have obtained this result directly by using the same reasoning as in the derivation of (2). But Eq. (3) is much more instructive, as it gives us also *conditions* under which the velocity of the star drops out.

For what is measured as *Bradley aberration is not precisely the relative aberration.* We don't simultaneously have two observers handy. What is measured is the difference in angles obtained by *one* observer in different states of motion, at different times. But this is the same thing as the relative aberration *provided* the velocity of the star is constant between measurements. Then the only velocity that counts is V_{21}, a *relative* velocity again. *If* the velocity of the star changed, we would obtain, instead of (3)

$$\tan\frac{\theta_2}{2} = \tan\frac{\theta_1}{2}\sqrt{\frac{(c-W_1)(c+V_1)}{(c+W_1)(c-V_1)}\frac{(c+W_2)(c-V_2)}{(c-W_2)(c+V_2)}}\,, \tag{4}$$

where now W_1 and W_2 are the velocities of the star, when it emitted the light rays reaching the observer when she had the velocities V_1 and V_2, respectively.

Sardin's device to measure "absolute speeds" replaces the star by a light source *moving along with the observer.* Thus the velocity of the source *changes* between successive observations at different observer speeds. Hence, the independence of Bradley aberration of the source velocity does not hold. But this is the central argument on which the working principle of the device is based. Therefore, the device will not work. In fact, we can calculate the aberration measured by the device from (4), because the velocities of the light source are known here. In any state of uniform motion we have $W_1 = V_1$, $W_2 = V_2$. The result $\tan\theta_2/2 = \tan\theta_1/2$ indicates that there will be no observable aberration. There is no need to abandon the relativity principle.

[1] Sardin G., *Measure of the absolute speed through the Bradley aberration of light beams on a three-axis frame*, Europhys. Lett. **53** (2001) 310 .

[2] Eisner E., *Aberration of Light from Binary Stars - a Paradox?*, Am. J. Phys. **35** (1967) 817.